

Battery Life Testing

1. A company tested the lifespan of 10 randomly selected batteries, recording an average time to failure of 9.5 hours with a standard deviation of 1.4 hours.

$$\alpha = 0.05$$

$$\alpha/2 = 0.025$$

$$\Rightarrow n = 10$$

$$\Rightarrow s = 1.4$$

$$\Rightarrow \bar{x} = 9.5$$

- a) Determine a 95% confidence interval for the population mean time to failure.

$$\Rightarrow \left(\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \right)$$

$$t_{\alpha/2, n-1} = 2.262$$

- b) Determine a 99% confidence interval for the population mean time to failure.

$$\alpha = 0.01$$

$$\alpha/2 = 0.005$$

$$t_{\alpha/2, n-1} = 3.25$$

\bar{x}
 μ
 s
 σ

n
 α
 s

population

s
 σ

2. A survey aims to estimate the average annual salary of employees in a city. A random sample of 500 workers reveals a mean salary of \$42,350 with a standard deviation of \$5,300.

- a) Determine a 95% confidence interval for the average salary of workers in this city.

$$\left(\bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}} \right)$$

$$Z_{\alpha/2} \Rightarrow 1.96$$

$$\alpha/2 = 0.025$$

$$\bar{x}, s \rightarrow \text{population}$$

$$\alpha = 1 - 0.95 \Rightarrow 0.05$$

- b) Determine a 98% confidence interval for the average standard deviation of workers in this city.

$$n = 500$$

$$s = 5300$$

$$\bar{x} = 42350$$

$$\sqrt{\frac{(n-1)s^2}{2}}$$

$$\chi^2_{\alpha/2, (n-1)}$$

$$\sqrt{\frac{(n-1)s^2}{2}}$$

$$\chi^2_{1-\alpha/2, n-1}$$

$$7.6324$$

$$36.1909$$

Water Bottle Volume Testing

4. A water bottle company checks that each bottle holds approximately 500 ml of water. A random sample of 12 bottles yields the following volumes (in ml):

498, 502, 499, 500, 497, 504, 501, 498, 500, 503, 497, 496. $\rightarrow \bar{x}$
 a) Determine a 95% confidence interval for the population standard deviation bottle volume using the t-distribution.

$n = 12$
 $s = 2.6391$
 $\bar{x} = 499.58$

$\alpha_2 = 0.025 \Rightarrow$

$\chi^2_{\alpha/2, n-1} = 21.9201$

$\chi^2_{1-\alpha/2, n-1} = 3.8158$

- b) Determine a 98% confidence interval for the population mean bottle volume using the t-distribution.

$\Rightarrow \left(\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \right)$

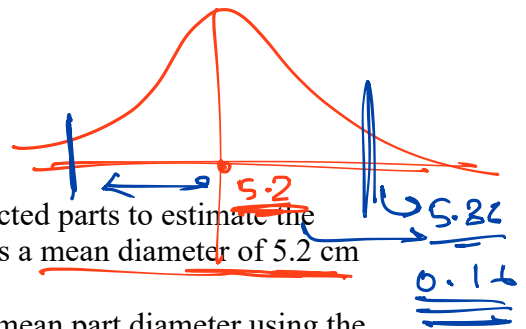
$t_{\alpha/2, n-1} = 2.718$

Machine Part Variation

$\bar{x} = 5.2$
 $s = 0.3$
 $n = 15$

5. A manufacturer measures the diameter of 15 randomly selected parts to estimate the variability in the production process. The sample data yields a mean diameter of 5.2 cm and a standard deviation of 0.3 cm.

a) Determine a 95% confidence interval for the population mean part diameter using the t-distribution.



$\left(\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \right)$

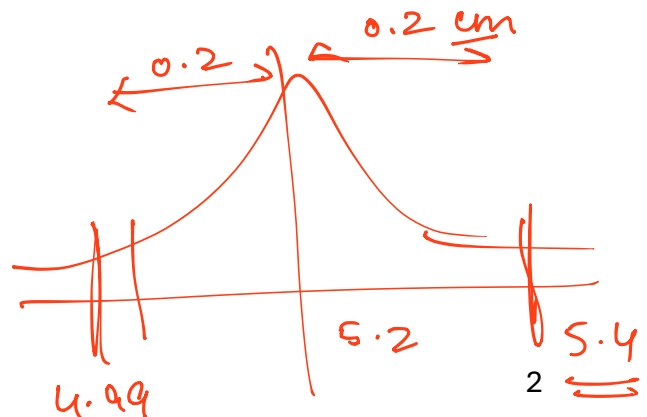
Ans

$(5.03, 5.36)$

- b) Determine a 98% confidence interval for the population mean part diameter using the t-distribution.

\Rightarrow

$(4.99, 5.40)$



Machine Part Standard Deviation

$$n=15$$

$$s=0.3$$

$$\alpha/2 = 0.01$$

\Rightarrow

$$\Rightarrow (0.2079, 0.5199)$$

6. A manufacturer measures the diameter of 15 randomly selected parts to estimate the variability in the production process. The sample data yields a standard deviation of 0.3 cm.
- a) Determine a 98% confidence interval for the population standard deviation.

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}}} \quad , \quad \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}}$$

- b) Determine a 95% confidence interval for the population standard deviation using the chi-square distribution.

Quality Control on Bottle Caps

7. A company inspects 18 randomly selected bottle caps and calculates a standard deviation of 0.2 cm for their widths.

- a) Determine a 99% confidence interval for the population mean of the cap.

- b) Determine a 95% confidence interval for the population standard deviation of the cap width.

Customer Satisfaction Survey

8. A company conducts a customer satisfaction survey for its new product. Out of 1,200 surveyed customers, 780 report that they are satisfied with the product.

a) Determine a 90% confidence interval for the true proportion of satisfied customers.

$$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$n = 1200$$

$$\hat{p} = \frac{780}{1200} = 0.65$$

$$(0.6273, 0.6726)$$

$$\alpha/2 = 0.05$$

b) Determine a 95% confidence interval for the true proportion of satisfied customers.

$$\alpha/2 = 0.025$$

Proportion Testing with Various Sample Sizes

9. For each of the following sample sizes, determine a 95% confidence interval for the population proportion if the observed proportion (\hat{p}) is 0.75.

a) n = 500

$$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

b) n = 750

c) n = 1000

d) n = 1500

e) n = 2000